DULWICH COLLEGE | SINGAPORE |

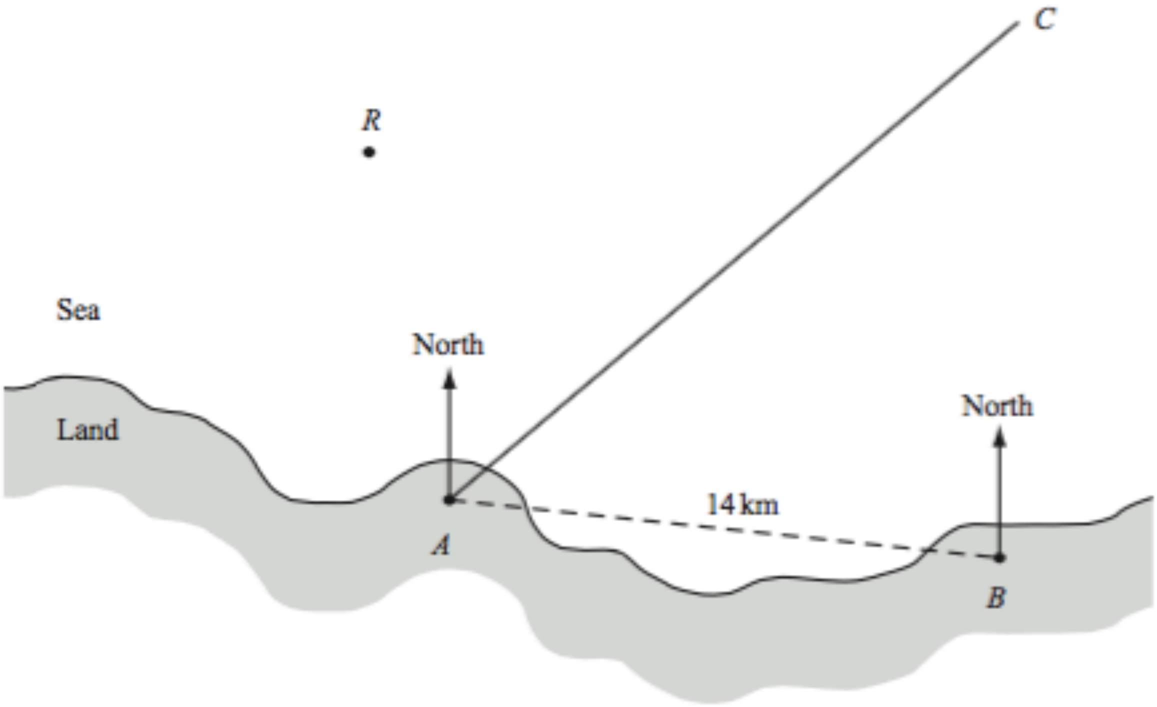
# Year 9 Examination
# Mathematics (Paper 4 - Extended)
# May 2017

Name:…………………………………………………………..............
Time allowed: 1 hour 30 minutes. Calculators allowed.

| **Marks** | | **Teacher comment:** |
|---|---|---|
| | % | |
| **Level/Grade** | | |

**Student reflection**

1




At midday, a ship is somewhere along the line from $A$ to $C$.

**(a)** By measuring an angle, write down the three figure bearing of the ship from $A$.

*Answer(a)* [2]

**(b)** The coastguard at $B$ sees the ship on a bearing of $350^\circ$.

**(i)** On the diagram draw accurately the line showing a bearing of $350^\circ$ from $B$. [1]

**(ii)** On the diagram mark the position of the ship, $S$. [1]

**(c)** **(i)** Measure the length, in centimetres, of the line $AB$ on the diagram.

*Answer(c)*(i) cm [1]

**(ii)** The distance from $A$ to $B$ is 14 kilometres.
Calculate the scale of the drawing.
Give your answer in the form $1:n$.

*Answer(c)*(ii) 1: [2]

2.

**(a)** The technical data of a car includes the following information.

| Type of road | Petrol used per 100 km |
|---|---|
| Main roads | 9.2 litres |
| Other roads | 8.0 litres |

**(i)** How much petrol is used on a journey of 350 km on a main road? [1]

.........................................

**(ii)** On other roads, how far can the car travel on 44 litres of petrol? [1]

.........................................

**(iii)** A journey consists of 200 km on a main road and 160 km on other roads.

**(a)** How much petrol is used? [2]

.........................................

**(b)** Work out the amount of petrol used per 100 km of this journey. [1]

.........................................

**(b)** A model of a car has a scale of 1 : 25.

**(i)** The length of the car is 3.95 m.
Calculate the length of the model.
Give your answer in centimetres. [3]

.........................................

**(ii)** The painted surface area of the model is 128 $cm^2$.
Calculate the painted surface area of the car, giving your answer in square centimetres.

.............................................(2)

3.

Luis and Hans both have their birthdays on January 1st.
In 2002 Luis is 13 and Hans is 17 years old.

**(a)** Which is the next **year** after 2002 when both their ages will be prime numbers?

*Answer (a)* ........................................................

**(b)** In which **year** was Hans twice as old as Luis?

*Answer (b)* ........................................................

**(a)** **(i)** Factorise.

$$2x^2 - x - 1$$

**(b)** Write down one possible value of $x$ that satisfies each inequality.

**(i)** $2 < \sqrt{x} < 3$

*Answer* $x =$ ...................................... [1]

**(ii)** $-1 < x^3 < 0$

*Answer* $x =$ ...................................... [1]

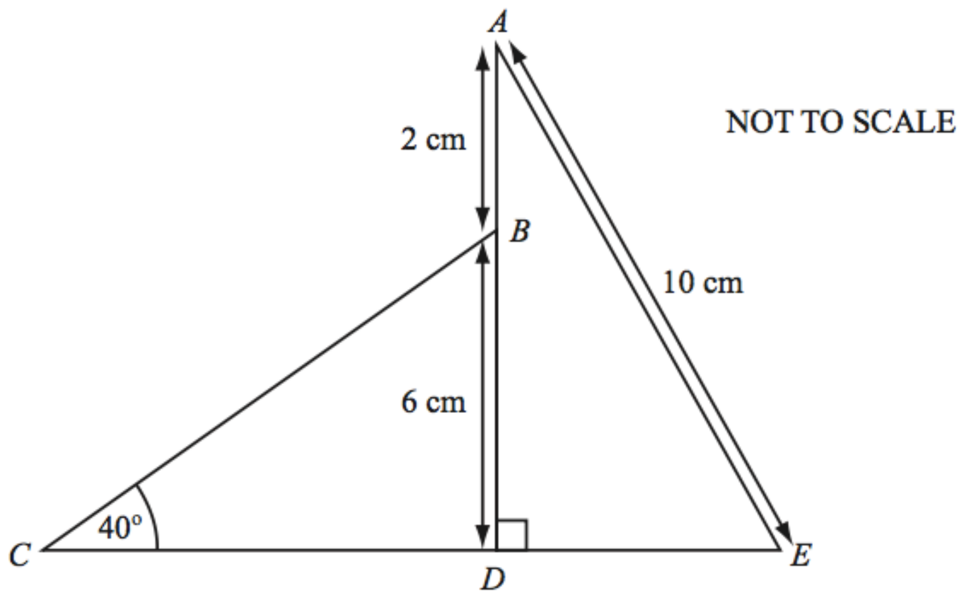
(c)

24 is a common factor of 96 and the integer $n$.

Given that $n$ is less than 96, find the largest possible value of $n$.

Answer n = ......................................(1)

4.




On the above diagram, $AB = 2$ cm, $BD = 6$ cm, $AE = 10$ cm, angle $BCD = 40°$ and angle $BDE = 90°$.

**(a)** Write down the length of *AD*.

*Answer(a) AD =* cm [1]

**(b)** **Calculate** the length of *DE*.

*Answer(b) DE =* cm [2]

**(c)** **Calculate** the size of angle *AED*.

*Answer(c)* angle *AED =* [2]

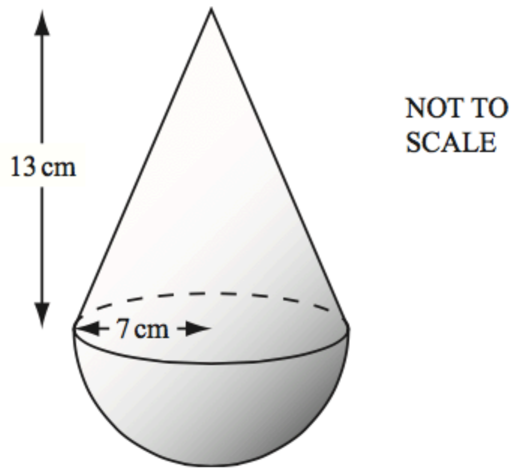
**(d)** **Calculate** the length of *CD*.

*Answer(d) CD =* cm [3]

**(e)** Find the length of *CE*.

*Answer(e) CE =* cm [1]

5




The diagram shows a solid made up of a hemisphere and a cone.
The base radius of the cone and the radius of the hemisphere are each 7 cm.
The height of the cone is 13 cm.

**(a)** **(i)** Calculate the total volume of the solid.

[The volume of a hemisphere of radius $r$ is given by $V = \frac{2}{3}\pi r^3$.]

[The volume of a cone of radius $r$ and height $h$ is given by $V = \frac{1}{3}\pi r^2 h$.] [2]

**(ii)** The solid is made of wood and 1 $cm^3$ of this wood has a mass of 0.94 g.
Calculate the mass of the solid, in kilograms, correct to 1 decimal place. [3]

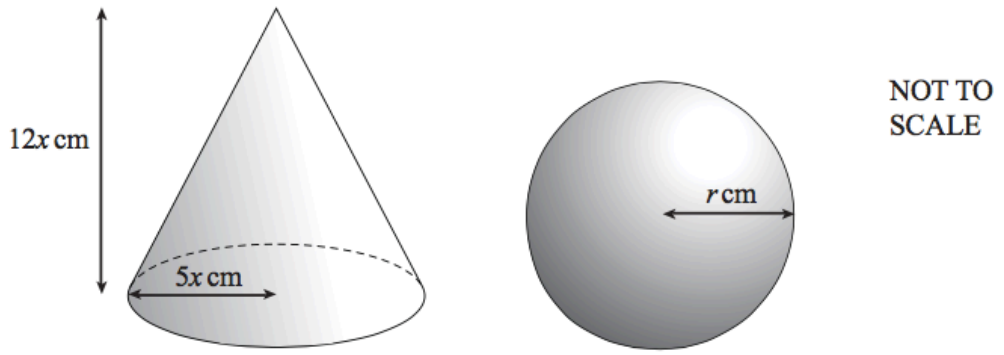
**(b)** Calculate the curved surface area of the cone.
[The curved surface area of a cone of radius $r$ and sloping edge $l$ is given by $A = \pi r l$.] [3]

**(c)** The cost of covering all the solid with gold plate is $411.58.
Calculate the cost of this gold plate per square centimetre.

[The curved surface area of a **hemisphere** is given by $A = 2\pi r^2$.] [5]

(d)

The diagram below shows a solid circular cone and a solid sphere.




The cone has the same **total** surface area as the sphere.
The cone has radius $5x$ cm and height $12x$ cm.
The sphere has radius $r$ cm.

Show that $r^2 = \frac{45}{2}x^2$.

[The curved surface area, $A$, of a cone with radius $r$ and slant height $l$ is $A = \pi rl$.]
[The surface area, $A$, of a sphere with radius $r$ is $A = 4\pi r^2$.]

( 4 )

6 Find the $n$th term of each of the following sequences.

**(a)** 21, 17, 13, 9, 5, .........

*Answer(a)* ................................................ [2]

**(b)** 3, 6, 12, 24, 48, .........

*Answer(b)* ................................................ [2]

**(c)** $\frac{1}{4}$, $\frac{4}{5}$, $\frac{9}{6}$, $\frac{16}{7}$, $\frac{25}{8}$, .........

Answer (c) ........................................................ (2)

**(b)** A sequence of diagrams is formed by drawing equilateral triangles with sides that measure one centimetre.

Diagram 1 Diagram 2 Diagram 3

Diagram 1 has 3 one-centimetre lines.
Diagram 2 has 9 one-centimetre lines.

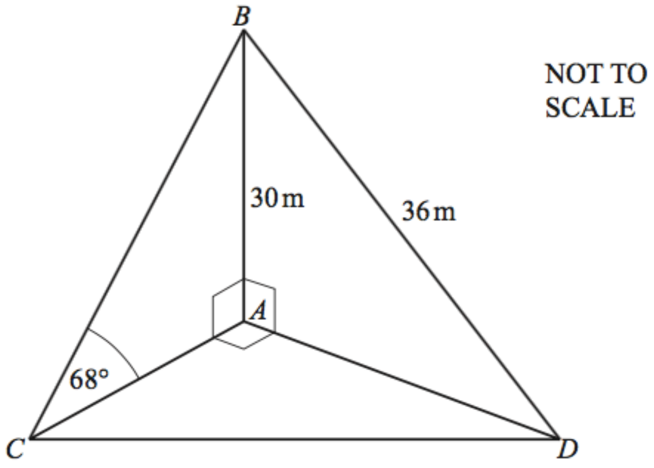
The expression for the **total** number of one-centimetre lines needed to draw **all of the first *n* diagrams** is

$$an^3 + bn^2 + n.$$

Find the value of $a$ and the value of $b$.

a = .......................... b = ............................... (4)

7




*AB* is a vertical tower of height 30 m.
*BC* and *BD* are straight wires attached to *B*.
*A*, *C* and *D* are on horizontal ground with *C* due west of *D*.
Angle *BCA* = 68° and *BD* = 36 m.

**(a)** Calculate *AD*.

*AD* = ........................................................... m [3]

**(b)** Calculate *AC* and show that it rounds to 12.1 m, correct to 3 significant figures.

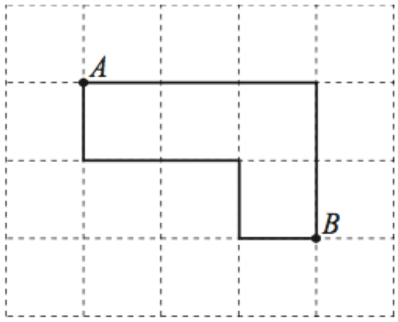
[3]

**(c)** Calculate the bearing of *A* from *D*.

........................................................... [3]

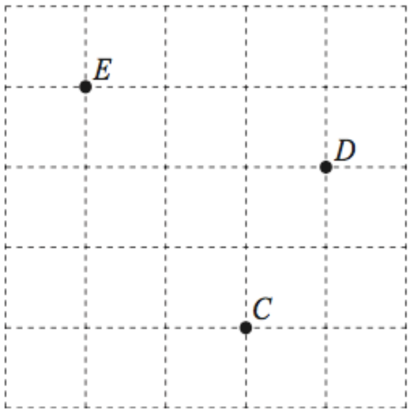
8. INVESTIGATION

A taxicab has to travel from $A$ to $B$. In taxicab geometry, to go from $A$ to $B$, you must only go along gridlines and take a shortest route.




The diagram shows two of the possible shortest routes from $A$ to $B$.
The taxicab distance $AB$ is 5.

**(a)**

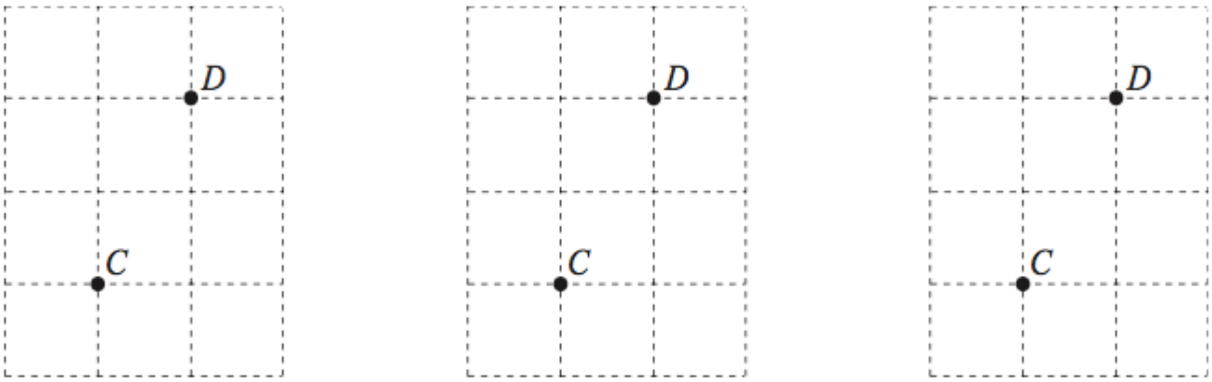



For this grid, write down the taxicab distance $CD$ and the taxicab distance $DE$.

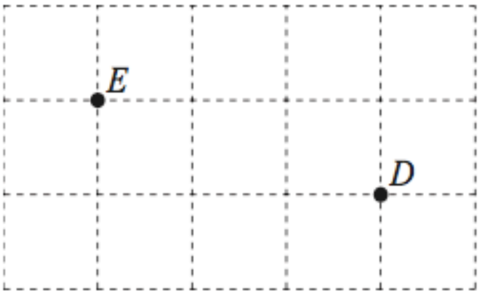
Taxicab distance $CD$ ..............................................

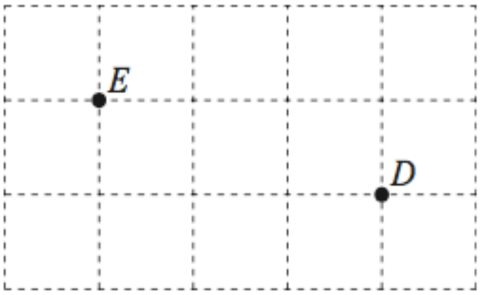
Taxicab distance $DE$ ..............................................

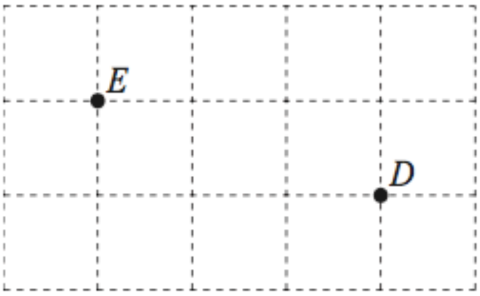
**(b)** On the grids below, show the three possible shortest routes from $C$ to $D$.
Remember, you must only go along gridlines.

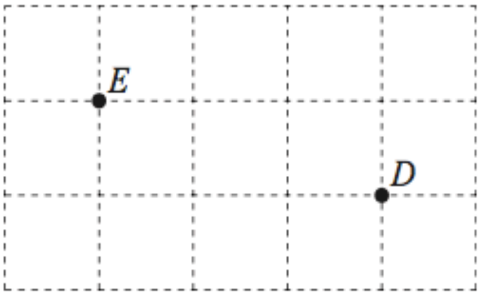


**(c)** On the grids below, show all of the possible shortest routes from $D$ to $E$.
Draw one route on each grid.



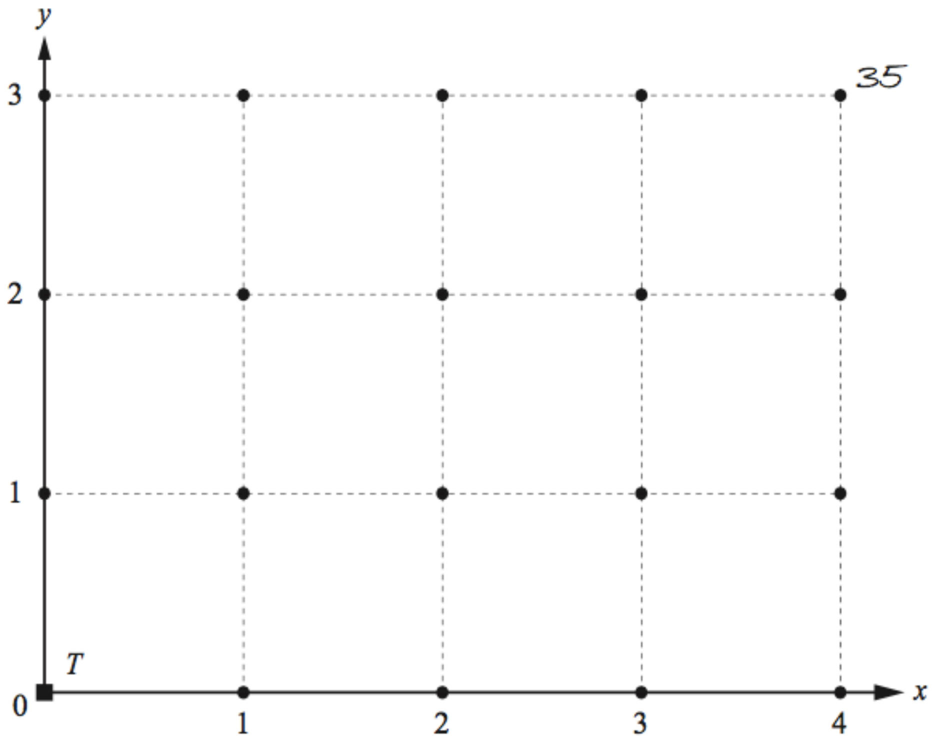

**(d)** **(i)** On the grid below, plot two points with taxicab distance equal to 5 and only one possible shortest route between them.

**(ii)** On the answer grid below, plot two points with taxicab distance **not** equal to 6 and exactly six possible shortest routes between them.
You may use the first grid for your working.

*Working grid*

*Answer grid*

**2** The taxicab is based at $T$, (0, 0) on the grid.
Possible destinations are marked •.
There are 35 possible shortest routes from $T$ to (4, 3).




**(a)** Write beside each destination on the $x$-axis and the $y$-axis, the **number** of shortest routes from $T$.

**(b)** There are three shortest routes from $T$ to destination (1, 2).
Each shortest route goes through either (0, 2) or (1, 1).

Explain how the number of shortest routes to (0, 2) and to (1, 1) can be used to find the number of shortest routes to (1, 2).

..........................................................................................................................................

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**(c)** Write beside each destination on the grid the **number** of shortest routes from $T$.

**(d)** There are 120 shortest routes from $T$ to destination (7, 3).

How many shortest routes are there from $T$ to (6, 3) and what is this taxicab distance?

Number of shortest routes ......................

Taxicab distance ....................................